



Simulation

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Introduction

- Simulation models “describe” how a system operates.
- They are referred to as “cause-and-effect” models.
- They describe the state of the system in response to various inputs but give no direct measure of what decisions should be taken to improve the performance of the system.

Introduction

- The simulation is problem solving technique:
 - a) development of a model of the system;
 - b) operation of the model (i.e. generation of outputs resulting from the application of inputs); and
 - c) observation and interpretation of the resulting outputs.

Introduction

- Major components of a simulation model are:
 - *Input*: quantities that “drive” the model (in water resources engineering models for example a principal input is the set of streamflows, rainfall sequences, pollution loads, water and power demands, etc.).
 - *Physical Relationships*: mathematical expression of the relationship among the physical variables of the system being modeled (continuity, energy conservation, etc.).

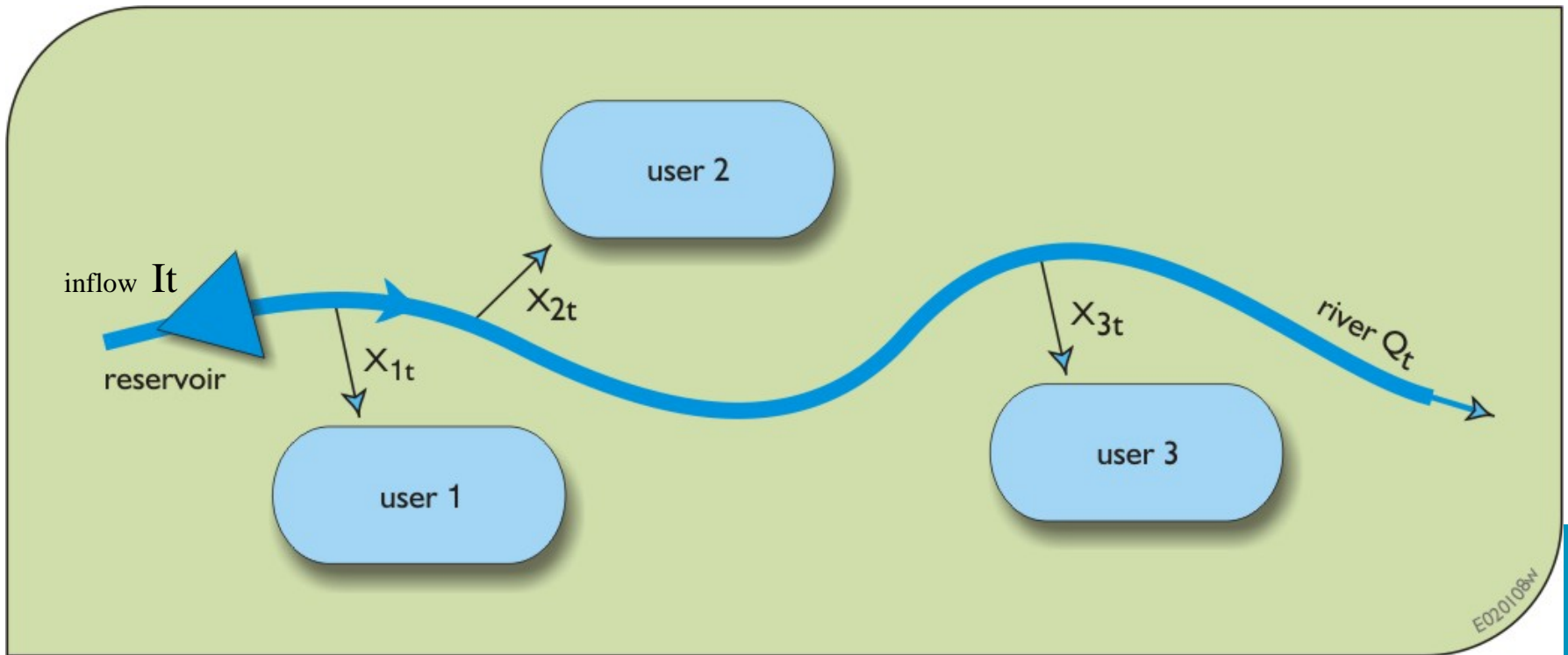
Cont.

- *Nonphysical Relationships*: those that define economic variables, political conflicts, public awareness, etc.
- *Operation Rules*: the rules that govern operational control.
- *Outputs*: the final product of operations on inputs by the physical and nonphysical relations in accordance with operating rules.

Example

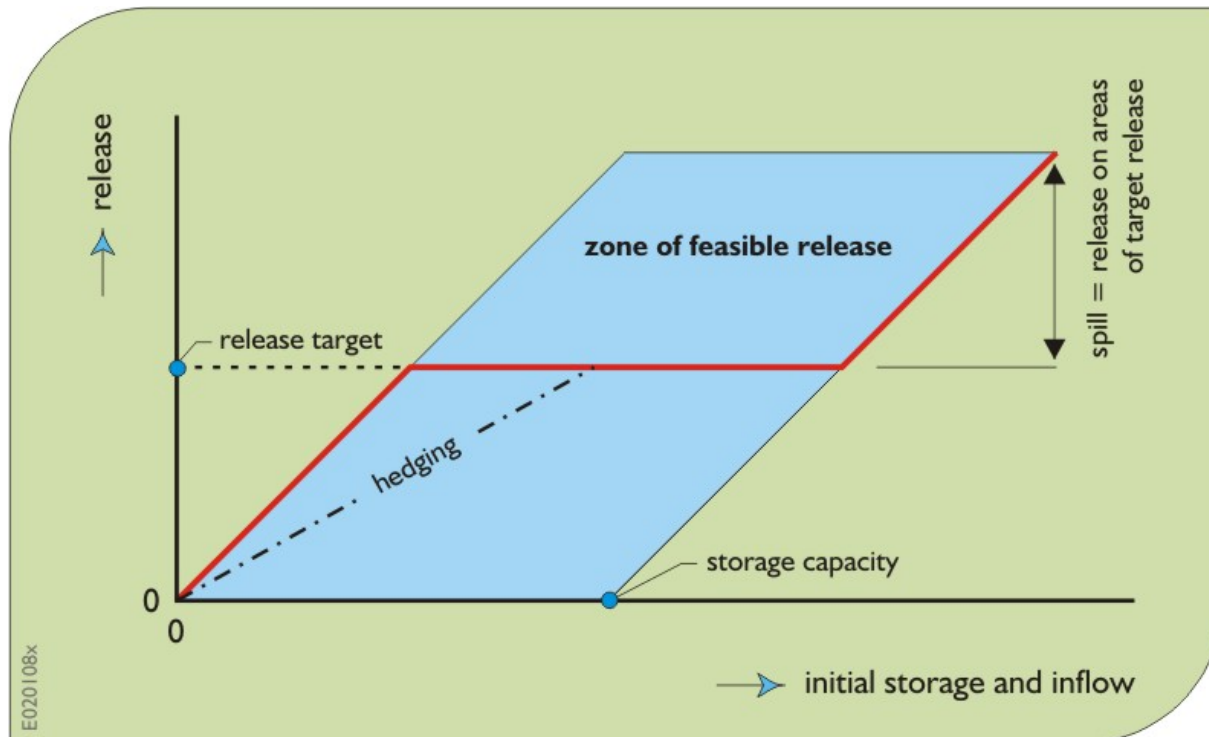
- Consider the case of a potential reservoir releasing water to downstream users. A reservoir and its operating policy can increase the benefits each user receives over time by providing increased flows during periods of otherwise low flows relative to the user demands. Of interest is whether or not the increased benefits the water users obtain from an increased flow and more reliable downstream flow conditions will offset the costs of the reservoir.

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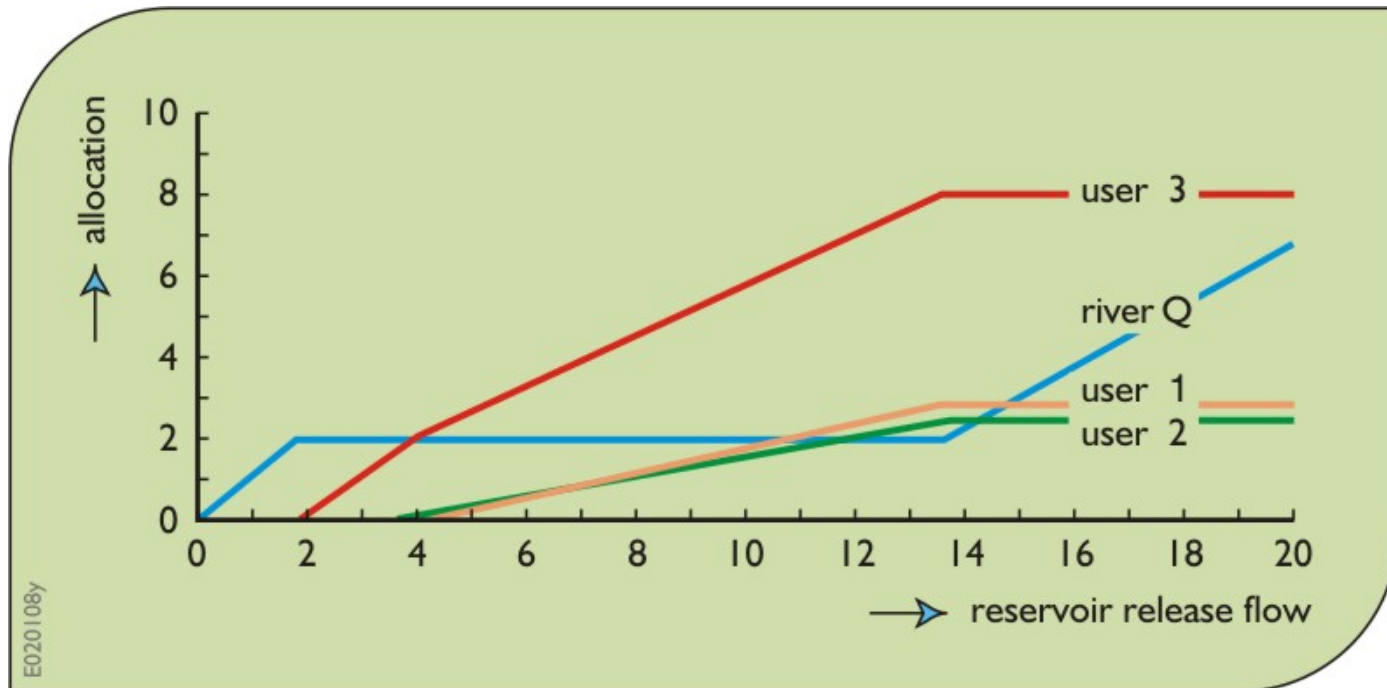
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- First we need to know the reservoir operating policy



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- Then we need to know water allocation policies



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- Develop a flow diagram of the reservoir-allocation system simulation process.

$I(t)$ - inflow

$S(t)$ - initial storage volume

$R(t)$ - reservoir release

$X_i(t)$ - allocation to user i

$Q(t)$ - allocation to stream

$B_i(t)$ - benefit for user i

t - time period

An alternative!

- Let's look again at the guts of the reservoir problem:

$$\mathbf{S(t+1) - S(t) = I(t) - R(t)}$$

i.e., "change in storage = inflow – release for demand"

- A mass balance over one period:

$$\Delta \mathbf{S} = \mathbf{Inflow} - \mathbf{Release}$$

- In general:

$$\Delta \mathbf{S} / \Delta \mathbf{T} = \mathbf{Inflow} - \mathbf{Release}$$

where ΔT = time step

- This is a "difference equation", which would be a *differential equation* if ΔT were to approach zero (dt)

An alternative!

- Many natural phenomenon are described by differential equations – e.g., diffusion, advection, wave propagation, ...
- Simulation is the creation of a mathematical model of a natural phenomenon and the execution of that model by a computer.
- Simulation modeling by computer usually implies a numerical (rather than analytical) approach to solving the underlying equations.
 - time is discretized – in our case, on a “**continuous**” basis, meaning broken into **regular** time steps.
 - “**discrete**” models usually mean ones with **irregular** time steps.

System Dynamics Simulation

- System Dynamics

A rigorous method of system description, which facilitates feedback analysis via a simulation model of the effects of alternative system structure and control policies on system behavior

- Management science – MIT (Forrester)

System Dynamics Simulation

- System Dynamics simulation approach relies on understanding complex inter-relationships existing between different elements within a system.
- This is achieved by developing a model that can simulate and quantify the behavior of the system.
- Simulation of the model over time is considered essential to understand the dynamics of the system.

System Dynamics Definitions

- A *system* is defined as a collection of elements which continually interact over time to form a unified whole.
- The underlying pattern of interactions between the elements of a system is called the *structure* of the system.
- The term *dynamics* refers to change over time.

System Dynamics Definitions

- A system's structure determines the system's behavior.
- System Dynamics links the behavior of a system to its underlying structure.
- System Dynamics can be used to analyze how the structure of a system can lead to the behavior which the system exhibits.

SD simulation process

- Identification of variables
- Causal diagram
- Stock and flow diagram
- Equations
- Simulation

Causal loop diagram

- Notation for representing system structures:
 - Feedback loop (a causal diagram)
 - Circle or loop of cause-effect relationships

A Shift of Mind

Filling a glass of water.

- From the linear point of view:

I am filling a glass of water.

- However, as we fill the glass:

We are watching the water level.

We monitor the gap between the level and our goal, the desired water level.

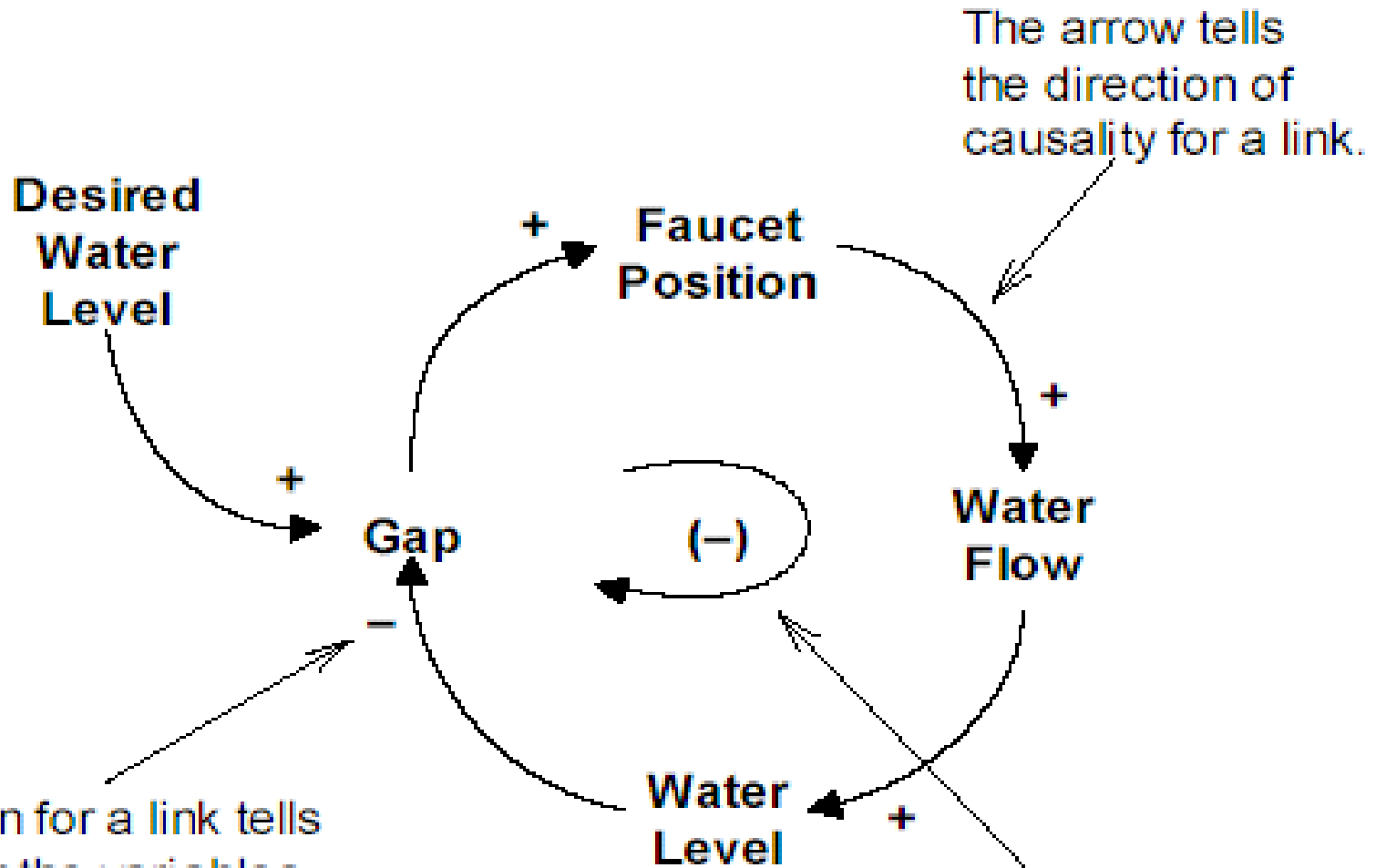
We are adjusting the flow of water.

A Shift of Mind

- Water regulation system
 - Variable 1 – the desired water level
 - Variable 2 – the water level
 - Variable 3 – the gap between the two
 - Variable 4 – the faucet position
 - Variable 5 – the water flow
- Organized in a circle or loop

CAUSAL LOOP DIAGRAM

[Filling a glass of water]



The arrow tells the direction of causality for a link.

The sign for a link tells whether the variables at the two ends move in the same (+) or opposite (-) directions.

The sign for a loop tells whether it is a positive (+) or negative (-) feedback loop.

Causal loop diagrams

- Arrows indicate causal influence



Causal loop diagrams

- Positive and negative signs show the direction of causality
 - + direct adds to
 - - inverse subtracts from

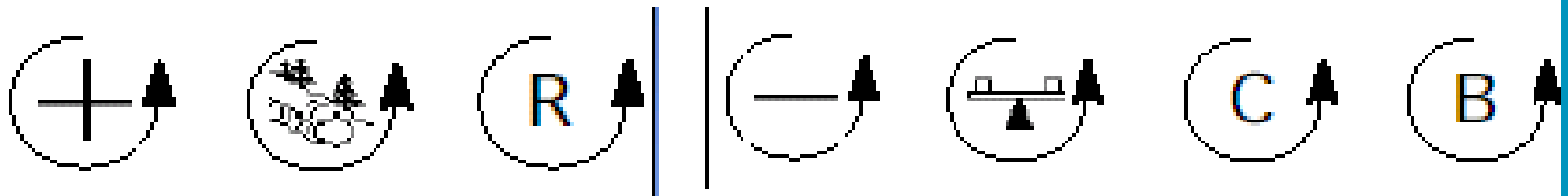


Two kinds of feedback loops

■ Positive

- Self-Reinforcing
- Growth producing
- Destabilizing
- Accelerating
- Even number of –'s

■ Symbolized by



■ Negative

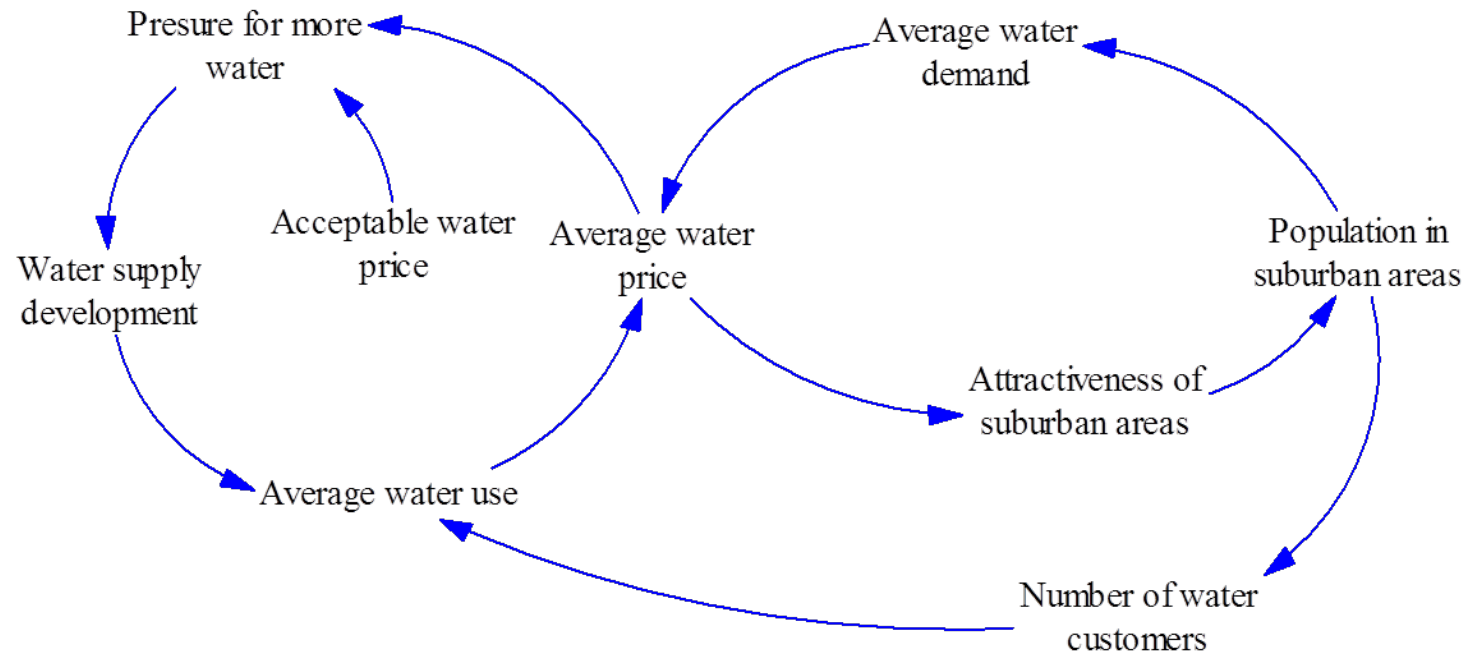
- Counteracting
- Goal seeking
- Stabilizing
- Balancing
- Odd number of –'s

■ Symbolized by

Formulation example

In water supply system capacity expansion and water demand problem below: (i) assign polarities to each of the causal links; (ii) assign polarities to each of the feedback loops; (iii) write a brief but insightful paragraph describing the role of the feedback loops in your diagram. Don't describe every link in your diagram (assume your figure and its polarities take care of that), talk mainly about the loops.

Formulation example



Stock and flow diagram

- Conversion of causal relationships into mathematical model
- Use of SD development tools (Vensim, STELLA, Powersim,...)

Basic building blocks of SD

- Levels (Stocks), Rates (Flows), Auxiliary variables and Arrows
- Essential building blocks
- Represent the way dynamic systems really work

The STOCK

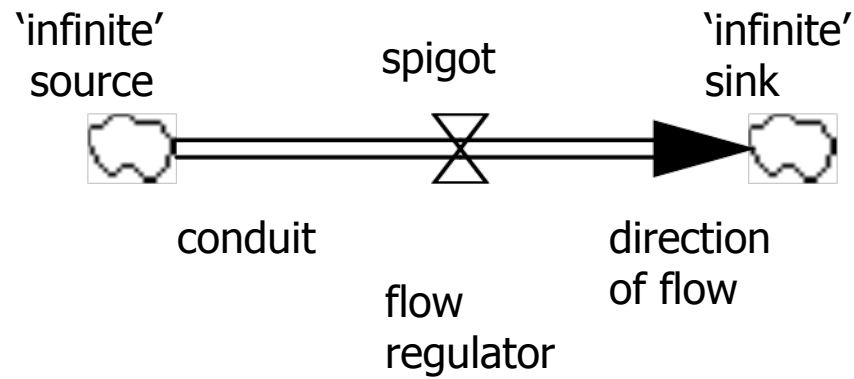


- Stocks represent *accumulations* (physical and non-physical).
- Stocks represent the 'traces' left by activities.
- The material in a stock exists at a point in time.
- Stock - level persist when activities cease.
- Examples: water, predators, CO₂, frustration, knowledge, vegetation,....

The FLOW

- Flows represent *activities* or *actions*.
- Flows occur over time.
- Flows:
 - Fill and drain stocks
 - Transport staff (physical and non-physical)
 - Can change 'instantaneously'
- Examples: eating, learning, runoff, communicating,....

The FLOW



Arrows

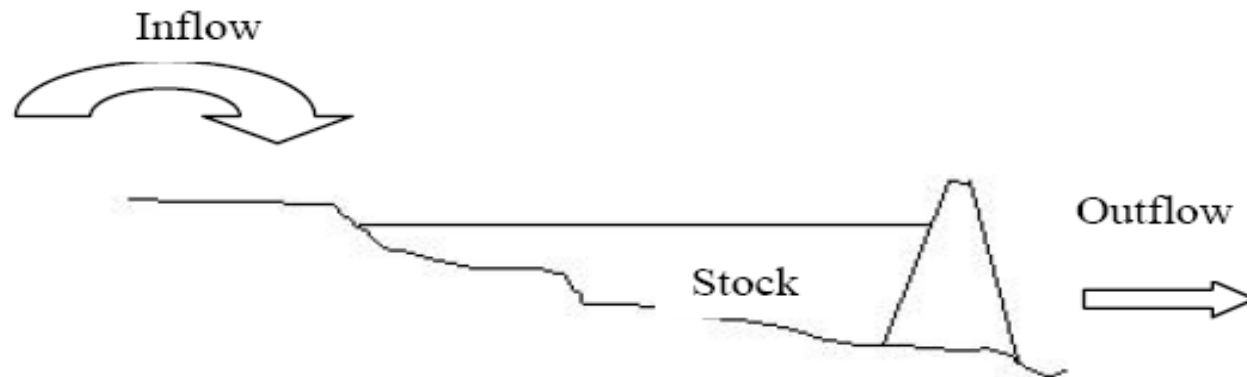


- Radiate signals that serve as inputs to decisions or actions.
- Arrows serve as inputs, rather than inflows, and outputs, rather than outflows.
- They link stocks and flows (and flows to flows), in order to generate actions.

Auxiliary variables

- They modify the activities (within the system)
- They transform inputs into outputs
- They represent information or material quantities
- They break out the detail of the logic
- They do not accumulate (their value is recalculated in each time step)
- They can be used for external inputs

System Dynamics Simulation



System Dynamics Simulation

$$Stock(t) = \int_{t_0}^t [Inflow(s) - Outflow(s)] ds + Stock(t_0)$$

$$\frac{d(Stock)}{dt} = Inflow(t) - Outflow(t)$$

System Dynamics Simulation

- If we assume constant flows during the time interval:

$$S_{t+dt} = S_t + dt \times (I_t - O_t)$$

- Basic technique known as Euler integration
- As the time step gets smaller the accuracy of Euler integration gets higher so in the limit

$$\lim_{dt \rightarrow 0} \frac{S_{t+dt} - S_t}{dt} = \frac{dS}{dt} = (I_t - O_t)$$

- Continuous time differential integration

System Dynamics Simulation

- The only difference between the numerical and analytic solution of the underlying differential equation system is the size of dt .
 - The differential equation uses an infinitesimal, a true instant.
 - Digital computers use discrete steps and a finite time step.
- The use of a finite time step and resulting approximations of flows over the interval introduces error, known as integration error or dt error.
- This error depends on how quickly the flows change relative to the time step.
- The faster the dynamics of the system, or the longer the dt , the larger the integration error.

System Dynamics Simulation

- How we select dt ?
 - Select a time step for your model that is a power of 2, such as 2, 1, 0.5, 0.25, etc.
 - Make sure your time step is evenly divisible into the interval between data points.
 - Select a time step one-fourth to one-tenth as large as the smallest time constant in your model.
 - Test for integration error by cutting the time step in half and running the model again. If there are no significant differences, then the original value is fine. If the behavior changes significantly, continue to cut the time step in half until the differences in behavior no longer matter.

VENSIM

- Short introduction

Example

- Red Reservoir problem (Section 8.2.7)

The Red River Valley has ideal growing conditions for several different types of vegetables, but very little rain. Federal funds were allocated for the construction of the Red Dam in a gorge on the Red River. This dam, together with the Red River Valley Irrigation Project, established an extensive irrigation system throughout the valley, and in the 20 years since the completion of the dam and irrigation system, a prosperous agricultural community has developed there.

■ ■ ■

- The *Inflow* to the Red Reservoir behind the Red Dam is not under our control.
- The amount of water in the reservoir is represented by the stock variable *Reservoirvolume*.
- All releases from the reservoir flow into the Red River Valley where the water is primarily used for irrigation of agricultural land. The amount of water available for irrigation at any time is also represented as a stock variable and named *Irrigation supply*.
- Water is consumed from the Red River Valley in a variety of ways, including evapotranspiration from plants, evaporation and drainage. The flow variable used to describe all of these losses is called *Drainage*. This drainage is not under the control of the Red Dam operator.
- Thus, there is only one decision variable, the *Release* through the Red Dam. **We shall examine decision policies for managing releases through the Red Dam for use in the Red River Valley**

VENSIM

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■ Data:

- The dam impounds water from a substantial stretch of the Red River, and the average net annual impoundment, after taking into account evaporation losses, is 0.5 million cubic metres (mcm).
- Standard operating procedure at the Red Dam is to maintain a long-term average of 1 mcm of water behind the dam in the Red Reservoir. However, the actual amount of water in the reservoir may vary over the short term depending on rainfall and other conditions.
- Not surprisingly, agriculture has expanded in the Red River Valley to consume 0.5 mcm per year of water.
- More specifically, the irrigation supply within the valley has 1 mcm of water accessible for agricultural use, and 50 % of this is consumed each year.

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- Reservoir operating rule
 - Any release rule has to consider both the *Reservoir volume* and the *Irrigation supply*.
 - Maintain sufficient flow to meet the irrigation demand in the Red River Valley, while providing that there is sufficient reserve in the Red Reservoir to meet the needs for irrigation during a drought period.
 - The quantity of water in the reservoir should not be so large that a sudden increase in inflow might lead to spill.
 - The *Reservoir volume* should not be insufficient to provide for irrigation if a drought occurs.
 - In order to prevent flooding in the Red River Valley during high flows and provide secure irrigation during low flows, we maintain a constant value for the *Irrigation supply*.

...

- Weighted-average reservoir model
 - If there is more water in the reservoir than the target, then the release should be increased, while if there is less water than the target, the release should be decreased.
 - Same for irrigation supply.
 - Different weights are assigned to meeting each goal depending on their relative importance.

VENSIM

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■ Equations

SAVEPER = TIME STEP – year

INITIAL TIME = 0

FINAL TIME = 4

TIME STEP = 0.01

Inflow = Long term average inflow + Test variation

Long term average inflow = 0.5 mcm

Drainage = 0.5 Irrigation supply

Irrigation supply = INTEG(Release – Drainage, Irrigation supply target)

*Irrigation supply target = 2 Long term average inflow
= 2 0.5 = 1 mcm*

Reservoir volume = INTEG(Inflow – Release, Reservoir Target)

Reservoir target = 2 Long term average inflow

Reservoir weight = 0.5

...

■ Equations

*Release = Long term average inflow +
Reservoir weight \times [(Reservoir volume – Reservoir target)/Time to
adjust reservoir volume] +
(1 – Reservoir weight) \times [(Irrigation supply target – Irrigation supply)/Time
to adjust irrigation supply]*

Time to adjust reservoir volume = 0.5

Time to adjust irrigation supply = 0.05

Test variation = STEP (0.1, 0.5)

VENSIM

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■ Simulation

- Run 1 Reservoir weight = 1
Irrigation supply weight = 0
- Run 2 Reservoir weight = 0.5
Irrigation supply weight = 0.5
- Run 3 Reservoir weight = 0
Irrigation supply weight = 1

