



# Restoration resource allocation model for enhancing resilience of interdependent infrastructure systems



Chao Zhang<sup>a,b</sup>, Jing-jing Kong<sup>c,d,\*</sup>, Slobodan P. Simonovic<sup>d</sup>

<sup>a</sup> Research Center of Modern Service Science and Technology, Shanghai University of Finance and Economics, Shanghai, China

<sup>b</sup> Shanghai Key Laboratory of Financial Information Technology, Shanghai University of Finance and Economics, Shanghai, China

<sup>c</sup> School of Civil Engineering, Shanghai Normal University, Shanghai, China

<sup>d</sup> Department of Civil and Environmental Engineering, The University of Western Ontario, London, ON, Canada

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## ABSTRACT

Enhancing the resilience of infrastructure systems is critical to the sustainability of the society against multiple disruptive events. This paper develops an approach for allocating restoration resources to enhance resilience of interdependent infrastructure systems. According to Inoperability Input–Output Model, a resilience metric for infrastructure systems is developed, in which the performance loss of infrastructure systems resulting from a disruptive event is measured in economic loss and inoperability. Model for determining the optimal infrastructure restoration resources allocation is proposed with the objective of maximizing resilience. Infrastructure interdependence is modeled by the Dynamic Inoperability Input–Output Model (DIIM), which is an accepted economic model for describing the interconnected relationship of industry sectors. To investigate the utility of the restoration resource allocation model, numerical analysis is conducted with an example derived from the data provided by the US Bureau of Economic Analysis. The results show that: (1) the optimal restoration resource allocation varies with the resource budget; (2) for a specific disruptive event, there exists an optimal resource budget which can minimize the sum of restoration cost and the performance loss of infrastructure system; and (3) the significance of factors such as initial inoperability of infrastructure systems on the optimal allocation. The proposed model can assist the decision makers in (i) better understand the effects of resource allocation, and (ii) deciding which allocation strategies should be used following a disruptive event.

## 1. Introduction

Modern society relies on the continuing services of infrastructure systems, e.g. transportation system, power grid, water supply system, as the backbone of national economy, security, and health. Critical infrastructure systems are complex with interconnected structural elements and functions. Interdependency is the basic operational characteristic of infrastructure systems. However, the infrastructure systems are becoming more vulnerable due to failure propagation across systems through the interconnected elements (Buldyrev et al., 2010). Large-scale disruptive events affecting infrastructure, though infrequent, are extremely costly to a society (Fang et al., 2015). Typical examples include 2003 power outage in North America, 2005 Hurricane Katrina in the USA (Leavitt and Kiefer, 2006), and the 2008 snow disaster in South China (Hou et al., 2008). The economic loss induced by these events can be very high (up to billions of dollars). For example, the 2003 power outage in North America generated almost US \$6 billion loss. The resilience is an important characteristic of real-world

systems affected by disruptive events, which are (i) related to systems' abilities to perform their functions, (b) reduce the magnitude of impacts of disruptive events through their adaptive capacity (National Infrastructure Advisory Council, 2009), and (iii) recover to normal functions (Ouyang and Wang, 2015). Given the increasing impact of natural and man-made disasters on infrastructure systems, improving resilience of interdependent infrastructure system is of growing importance. This requires quantifying the resilience of interconnected systems and developing approaches for enhancing resilience.

Some studies have developed resilience metrics for single infrastructure systems based on two system performance curves during a specific time period: one is the real performance curve, recording system performance change under a disruptive event and restoration activities, and the other is the expected performance curve, giving system performance level without a disruptive event. The resilience is then quantified as the area between the two curves within a restoration period (Bruneau et al., 2003; Bruneau and Reinhorn, 2007), or the area between the two curves during a given time period during which

\* Corresponding author at: School of Civil Engineering, Shanghai Normal University, Shanghai, China.  
E-mail addresses: [zhang.chao@snufe.edu.cn](mailto:zhang.chao@snufe.edu.cn) (C. Zhang), [kjj121123@gmail.com](mailto:kjj121123@gmail.com) (J.-j. Kong).

multiple disruptive events may happen (Ouyang and Dueñas-Osorio, 2012), or a normalized area under the performance curve of a system during a disruptive event (Chang and Shinozuka, 2004). These metrics quantify the resilience of an infrastructure system to a disruptive event or a sequence of disruptive events based on their performance losses. The basic idea of the aforementioned resilience metrics has been extended in a number of ways, for example, applying the ratio of recovery at a given time to the loss in performance (Henry and Ramirez-Marquez, 2012), or a stochastic metric by taking uncertainty into account (Pant et al., 2014a, 2014b). A dynamic resilience metric was proposed based on the adaptive capacity of an infrastructure system and the level of system performance, which can provide more insight into system performance evolution from the beginning of a disruptive event until the full performance recovery (Simonovic and Peck, 2013; Simonovic, 2016). However, since it is difficult to quantify the performance of different infrastructure systems into one formulation, the literature on resilience metrics concentrates on the capacity of a single system. Since the protection and recovery from infrastructure system failures are complex practical problems, the resilience of infrastructure systems becomes a focus point for policy making. There is a need for study of resilience of interdependent infrastructure systems and enhancement strategies.

Restoration activities are essential for enhancing resilience of infrastructure systems (Heath et al., 2016). Under large scale disruptive events, through supply of physical and/or financial restoration resources to infrastructure managers, central or local governments will help restore performance of damaged infrastructure systems and mitigate the disastrous impacts (MacKenzie and Zobel, 2016). Optimal resource allocation among infrastructures at the system level is critical for resilience enhancement due to the budget limitations. In the literature, resource allocation models seek to answer the question of how to satisfy specific goals with limited resources within a given constraints (MacKenzie et al., 2016), and have been applied to analyze many policy-related problems (Petrovic et al., 2012; Shan and Zhuang, 2013a, 2013b). The objective of restoration resources allocation is to help expedite the recovery of infrastructure systems, with consideration of their damage magnitudes and interdependencies. This problem has not been addressed in the available literature and will be investigated in this research.

Interdependencies among infrastructure systems should be considered in the restoration resource allocation problems. The effects of interdependencies include propagation of effects from one infrastructure system to another (Rinaldi et al., 2001). Therefore, a disruptive event that directly impacts one infrastructure system can trigger indirect impacts to other systems. Further, the performance recovery processes of impacted infrastructure systems are also affected by interdependencies (Baroud et al., 2015). A variety of models have been proposed to analyze the interconnected relationships among infrastructure systems (Ouyang, 2014). Network based models and economic theory based models are most commonly used. Interdependent infrastructure systems are described as multilayer networks in network based models. The interdependencies between systems can be quantified and analyzed at component level (Wang et al., 2013; Ouyang and Wang, 2015). In comparison, economic theory based models usually use infrastructure system, or subsystem, as the smallest analysis unit, and analyze the interdependencies at system level (Haimes et al., 2005a, 2005b). In this study, Dynamic Inoperability Input-Output Model (DIIM), one of the economic theory based models, proposed by Haimes et al. (2005a, 2005b), is chosen to capture the recovery dynamics of interdependent infrastructure systems. Based on the interdependency matrix and initial disturbances caused by a disruptive event, the DIIM can calculate the economic losses and inoperabilities of interdependent infrastructure systems during the recovery process (Lian and Haimes, 2006).

The main contributions of the present research include: (i) development of an optimization model for determining the optimal

allocation of restoration resources to interdependent infrastructure systems. As interdependencies among infrastructure systems are of great importance in system recovery process, the effects of interdependencies are embedded into the model by the application of DIIM. (ii) Application of the model to an example derived from the data provided by the BEA (the US Bureau of Economic Analysis). The example demonstrates the utility of the model in decision making. The results show (i) how to allocate limited resources to interdependent infrastructure systems, (ii) what is the optimal level of recovery budget for a specific disruptive event, and (iii) the significance of various factors on the level of resource budget for a specific infrastructure system.

The paper is organized as follows. Section 2 develops a resilience metric for interdependent infrastructure systems. With the objective of maximizing resilience, Section 3 proposes a restoration resources allocation model for enhancing resilience of interdependent infrastructure systems. Section 4 provides a numerical method for solving the resource allocation model. Section 5 investigates the utility of the model through numerical analysis. Section 6 concludes.

## 2. Resilience of infrastructure systems

### 2.1. Resilience metric for single infrastructure system

From engineering-based point of view, infrastructure system resilience is derived from the change in system performance over time (MacKenzie and Zobel, 2016). The resilience model derived by MCEER (Multidisciplinary Center for Earthquake Engineering Research, Bruneau and Reinhorn, 2007) quantifies the resilience as the area under the system performance curve (describing system performance from the beginning of system disturbance until full system recovery shown as the area under system performance with restoration strategy from  $t_{DO}$  to  $t_{RE}$  in Fig. 1). In order for easy comparison among diverse systems, system resilience is measured as the ratio of the area under system performance with restoration strategy to the area under expected system performance from  $t_{DO}$  to  $t_{RE}$  (Zobel, 2011; Simonovic and Peck, 2013). Then the resilience of infrastructure  $\phi$  under a disruptive event is expressed as

$$\gamma_{\phi} = \frac{\int_{t_{DO}}^{t_{RE}} SP_{\phi}^A(t) dt}{\int_{t_{DO}}^{t_{RE}} SP_{\phi}^E(t) dt} = \frac{\int_{t_{DO}}^{t_{RE}} (SP_{\phi}^E(t) - SL_{\phi}(t)) dt}{\int_{t_{DO}}^{t_{RE}} SP_{\phi}^E(t) dt} = 1 - \frac{\int_{Rapidit_{\phi}} SL_{\phi}(t) dt}{\int_{Rapidit_{\phi}} SP_{\phi}^E(t) dt} \quad (1)$$

where  $t_{DO}$  is the occurrence time of a disruptive event,  $t_{RE}$  is the full recovery time of infrastructure system  $\phi$ ,  $SP_{\phi}^E(t)$  is the expected system performance level,  $SP_{\phi}^A(t)$  is the actual system performance,  $SL_{\phi}(t)$  is the difference between  $SP_{\phi}^E(t)$  and  $SP_{\phi}^A(t)$ . Rapidity refers to the capacity to meet priorities and achieve goals in a timely manner, which is measured by the duration of system performance recovery and expressed as  $Rapidit_{\phi} = t_{RE} - t_{DO}$ . Robustness refers to the ability of a system to withstand a given level of stress without suffering further degradation or loss of function. It is usually quantified as the minimum system performance under recovery process.

According to Eq. (1), system resilience is the proportion of the shaded area to the area under expected system performance. The level of robustness indicates that the infrastructure system is not totally damaged by a disruptive event but, without self-repairing capability, it could not recover to normal performance level. Since the robustness of an infrastructure system under a specific disruptive event is fixed (property of the system structure), the system resilience is determined by the restoration activities. In Fig. 1, the result of a restoration strategy  $i$  is illustrated as the shaded area. The different contributions of restoration strategy  $i$  and  $j$  to resilience could be measured by the difference in shaded area between system performance curves with the two restoration strategies.

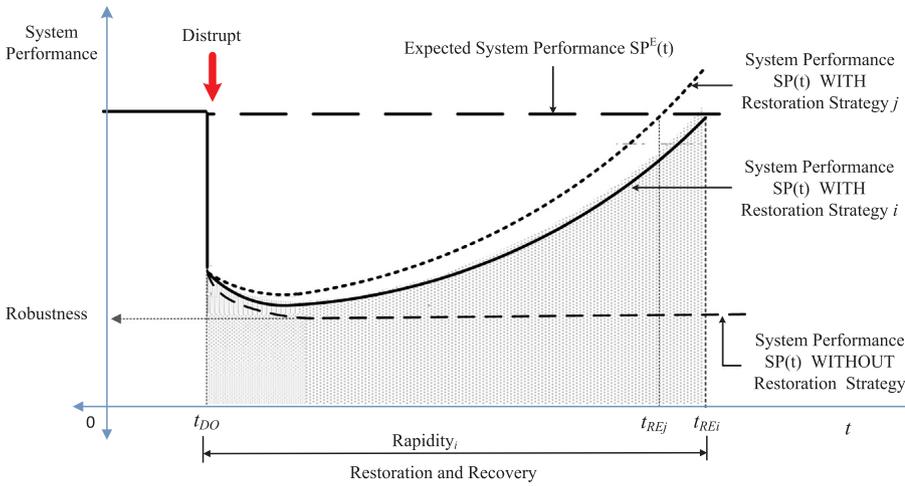


Fig. 1. Typical performance of an infrastructure system under a disruptive event.

### 2.2. Resilience metric for infrastructure systems

Infrastructure systems such as roads, electric power grid, water distribution network, function together as a “system of systems” (Satumtira and Dueñas-Osorio, 2010). As shown in Eq. (1), infrastructure system resilience is quantified using the loss of system performance. However, for different infrastructure systems, their performance is expressed in different units, such as [km] of blocked streets, [m<sup>3</sup>] of water distribution volume, [kw] of power transmission capacity, and [GB] of unavailable internet traffic volume. In the literature, the loss of system performance has been measured by the number of damaged components, efficiency of infrastructure service delivery, and impacted population.

To quantify the resilience of ‘system of system’, we express the performance loss of diverse infrastructure systems in the same form according to Inoperability input–output Model (IIM, Crowther and Haimes, 2005). The IIM is a risk-based extension of the Leontief input–output framework, and with various applications to industry sectors or infrastructure systems. The IIM focuses on the inoperability of systems due to perturbations resulting from disruptive events, and the negative consequences are measured in economic loss and inoperability (i.e., percentage of “dysfunctionality” relative to an ideal state). In Eq. (1), the term  $SL_{\phi}(t)$  can be expressed as

$$SL_{\phi}(t) = \alpha_{\phi} q_{\phi}(t) \quad (2)$$

where  $\alpha_{\phi} = SP_{\phi}^E(t)$  represents the expected system performance level in monetary units,  $q_{\phi}(t)$  represents the inoperability, which quantifies the proportional extent to which infrastructure  $\phi$  is not functioning in an as-planned manner at  $t$ .

For simplification, it is supposed a disruptive event occur at  $t = 0$ . Given  $N$  types of infrastructure systems, through Eqs. (1) and (2), resilience of “system of systems” could be obtained and expressed as

$$\gamma = 1 - \frac{\sum_{\phi=1}^N \int_{T^*}^{\infty} SL_{\phi}(t) dt}{\sum_{\phi=1}^N \int_{T^*}^{\infty} SP_{\phi}^E(t) dt} = 1 - \frac{\sum_{\phi=1}^N \int_{T^*}^{\infty} \alpha_{\phi} q_{\phi}(t) dt}{T^* \sum_{\phi=1}^N \alpha_{\phi}} \quad (3)$$

where  $\gamma$  is the resilience of infrastructure systems,  $T^* = \max_{\phi} \{rapidity_{\phi}\}$ , which implies the performance of every infrastructure systems has recovered to expected level after time period  $T^*$ .  $\alpha_{\phi} = SP_{\phi}^E$  is the expected performance level of infrastructure  $\phi$ .

### 3. Resource allocation model to enhance resilience

Considering the interdependencies among infrastructure systems, this section proposes a resource allocation model for enhancing resilience of systems.

### 3.1. Model of resource allocation

Following a disruptive event, restoration activities will be implemented to infrastructure systems to recover their performance. As shown in Fig. 1, during the recovery process, the change of system performance or inoperability rests on the restoration strategy, which is mainly determined by the restoration capacity applied. The restoration capacity includes the number of repair crews, available equipment and replacement components. Therefore, when considering the effect of restoration activities, resilience of infrastructure systems (see Eq. (3)) can be expressed by Eq. (4).

$$\gamma = 1 - \frac{\sum_{\phi=1}^N \alpha_{\phi} \int_0^{T^*} q_{\phi}(\beta_{\phi}, t) dt}{T^* \sum_{\phi=1}^N \alpha_{\phi}} \quad (4)$$

where  $\beta_{\phi}$  represents restoration capacity applied to infrastructure  $\phi$ .

For small-scale disruptive events, restoration activities are implemented by infrastructure management departments. Let  $h_{\phi}$  represent the basic restoration capacity of infrastructure  $\phi$ 's management department, then we have

$$0 \leq \beta_{\phi} \leq h_{\phi} \quad \forall \phi \in [1, N]. \quad (5)$$

Following a large-scale disruptive event, such as Hurricane Sandy in 2013, a large number of infrastructure systems might be severely damaged simultaneously. To mitigate the disastrous impacts, infrastructure management departments can obtain resource support from local or central government. The restoration capacity increase of an infrastructure system depends on the available resources. Therefore, following constraint can be drawn:

$$0 \leq \beta_{\phi} \leq h_{\phi} + f_{\phi}(g_{\phi}) \quad \forall \phi \in [1, N] \quad (6)$$

where  $g_{\phi}$  represents the restoration resource obtained from the government. Function  $f_{\phi}$  describes the effect of restoration resources to raise the restoration capacity. Without generality, the government's resources budget should be limited. Taking  $N$  types of infrastructure systems as a whole, the total of restoration resources should satisfy the following constraint:

$$\sum_{\phi=1}^N g_{\phi} \leq G \quad (7)$$

where  $G$  represents restoration resource budget. As the budget is limited, it is necessary to allocate the resources in an effective way. According to Eq. (4), the decision makers can improve resilience  $\gamma$  by solving following optimization problem:

$$\max_{g_1, \dots, g_\phi, \dots, g_N} \gamma = 1 - \frac{\sum_{\phi=1}^N \alpha_\phi \int_{T^*} q_\phi(\beta_\phi(g_\phi), t) dt}{T^* \sum_{\phi=1}^N \alpha_\phi} \quad (8)$$

$$s. t. \begin{cases} \beta_\phi = h_\phi + f_\phi(g_\phi), & \phi = 1, 2, \dots, N \\ \sum_{\phi=1}^N g_\phi \leq G \end{cases} \quad (9)$$

Solution  $g = (g_1, g_2, \dots, g_N)^T$ , which maximizes the objective function (8), is the optimal resource allocation strategy. The first constraint in Eq. (9) ensures the restoration capacities of infrastructure systems are fully applied. The second constraint is drawn from Eq. (7).

### 3.2. Model of infrastructure interdependencies

To analyze the recovery process of infrastructure systems, the DIIM introduced by Haimes et al. (2005a, 2005b) is chosen to capture the recovery dynamics of interdependent infrastructure systems. For  $N$  types of systems, the inoperability form of the DIIM is given in

$$\dot{q}(t) = K \times [A^* \times q(t) + c^*(t)] - q(t) \quad (10)$$

where vector  $q(t) = (q_1(t), q_2(t), \dots, q_N(t))^T$  denotes the inoperability of infrastructure systems at  $t$ . Vector  $c^*(t) = (c_1^*(t), c_2^*(t), \dots, c_N^*(t))$  represents the normalized degraded production or performance output to users (other than infrastructure systems) at  $t$ .  $N \times N$  matrix  $A^*$  represents the normalized interdependency matrix, in which every entry represents how much inoperability is contributed by the column system to the corresponding row system due to the interdependent nature of system interactions. Matrix  $K = \text{diag}(k_1, \dots, k_N)$ ,  $k_\phi$  denotes the recovery rate of infrastructure  $\phi$ , measures the capability of the system to recover from the disruptive event and reach a desired performance state. (Pant et al., 2014a, 2014b).

Eq. (10) is a linear first-order differential equation (Hindmarsh and Rose, 1984), given that condition  $q(0)$  represents vector of initial inoperability  $q(t)$  of infrastructure systems. The solution to Eq. (10) is given by Eq. (11) (Haimes et al., 2005a, 2005b)

$$q(t) = e^{-K(I-A^*)t} q(0) + \int_0^t K e^{-K(I-A^*)(t-z)} c^*(z) dz \quad (11)$$

If  $c^*(t)$  is stationary, does not change with time, Eq. (11) can be written as

$$q(t) = (I-A^*)^{-1} c^* + e^{-K(I-A^*)t} [q(0) - (I-A^*)^{-1} c^*] \quad (12)$$

If the performance output of infrastructure systems gives priority to the user demands, and the performance output to users does not change, that is  $c^*(t) = 0$ , Eq. (12) can be written as

$$q(t) = e^{-K(I-A^*)t} q(0) \quad (13)$$

Eq. (13) shows that the temporal evolution of inoperability  $q(t)$  depends on the initial value  $q(0)$ , normalized interdependent matrix  $A^*$ , and the recovery rate matrix  $K$ . The bounds from 0 to 1 are established for the elements of matrix  $K$ , which are determined by the consideration that the matrix  $K(I-A^*)$  has positive eigenvalues. This guarantees that the solutions of inoperability  $q(t)$  do not diverge, an important consideration for modeling a trajectory that converges towards a stable state. With this constraint on elements of  $K$ , during the recovery process following a disruptive event, the inoperability  $q(t)$  for infrastructure systems will gradually recover to 0 as the exponential term  $e^{-K(I-A^*)t} q(0)$  decays with time.

According to the meaning of parameters, we set the diagonal elements of matrix  $K$  as the restoration capacity of infrastructure systems, that is  $k_\phi = \beta_\phi$ . Suppose functions  $f_\phi$  in Eq. (9) are known and describe the effect of allocating resources to increase the recovery rate  $k_\phi$ . In general, the functional form for  $f_\phi(g_\phi)$  should have certain properties. First, the first order derivatives  $df_\phi(g_\phi)/dg_\phi$  should be greater than or equal to 0, which means the recovery rate  $k_\phi$  will increase if more resources are allocated to infrastructure  $\phi$ . Second, the second order

derivative  $d^2f_\phi(g_\phi)/dg_\phi^2$  should be less than or equal to 0, which signifies constant returns or marginal decrease in improvements as more resources are allocated. The first unit of resource allocated to the system should at least be as effective as the second allocated unit.

Given above constraints, we assume the allocated resources will increase the recovery rate according to the logarithmic function shown in Eq. (14), which is a frequent assumption in engineering risk problems (MacKenzie and Zobel, 2016; Dillon et al., 2005).

$$k_\phi = \beta_\phi = h_\phi + \ln(1 + u_\phi g_\phi) \quad (14)$$

where  $h_\phi$  represents the basic restoration capacity of infrastructure  $\phi$ 's management department as defined in Eq. (5);  $u_\phi > 0$  is cost-effectiveness parameter, describing the effectiveness of allocating resources to infrastructure  $\phi$ 's recovery rate;  $g_\phi$  represents the restoration resource obtained from the government.  $u_\phi$  can be assessed if  $g_\phi$ ,  $h_\phi$  and  $k_\phi$  are known or can be estimated, since  $u_\phi = (e^{k_\phi - h_\phi} - 1)/g_\phi$ . The value of  $u_\phi$  is always greater than or equal to 0 but has no upper bound. We expect  $u_\phi$  to be very small as large amount of resources are required to decrease the impacts of a large-scale disruptive event. The function shown in Eq. (14) is strictly increasing and marginally decreasing with respect to  $g_\phi$ .

This study considers the condition that the performance output of infrastructure systems gives priority to the user demands, and the performance output to users do not degrade after a disruptive event, that is  $c^*(t) = 0$ . Substituting the expression of  $q(t)$  in Eq. (13), and the expression of  $\beta_\phi$  in Eq. (14) into Eq. (8), an explicit form of the resource allocation model is written as:

$$\max_{g_1, g_2, \dots, g_N} Q = 1 - \frac{1}{T^* |\alpha|} \int_0^{T^*} \alpha e^{-K|I-A^*|t} q(0) dt \quad (15)$$

$$s. t. \begin{cases} K = \text{diag}(h_\phi + \ln(1 + u_\phi g_\phi)), \phi = 1, 2, \dots, N \\ \sum_{\phi=1}^N g_\phi \leq G \\ g_\phi > 0, \quad i = 1, 2, \dots, N \end{cases} \quad (16)$$

where vector  $\alpha = (\alpha_1, \dots, \alpha_N)$ . The solution  $g = (g_1, g_2, \dots, g_N)^T$  to above optimization problem is the optimal allocation of resources that maximizes resilience of infrastructure systems.

### 4. Solution method

The resource allocation model shown by Eqs. (15) and (16) can be solved with methods such as Lagrange multiplier (Everett, 1963). However, due to the computational complexity of exponential term  $e^{-K|I-A^*|t}$  in Eq. (15), we are proposing a numerical method for solving the model by applying genetic algorithm (GA), which is powerful stochastic search algorithm that has been successfully used in literature to solve optimization problems in infrastructure restoration (Xu et al., 2007; Ouyang and Wang, 2015). The procedures to search for an optimal solution to the optimization problem can be described by the following steps.

Step 1. Codes design. Express each solution to resource allocation by a genotype, vector  $e = (e_1, e_2, \dots, e_N)^T$ , subject to the following constraints:

$$\begin{cases} \sum_{i=1}^N e_i \leq 1 \\ e_i \geq 0 \end{cases} \quad (17)$$

Considering the variety of individuals, the genotypes of initial individuals are randomly generated according to constraint conditions.

Step 2. Compute the fitness value of each genotype. Transform objective function (15) into discrete-time version of fitness function:

$$Q = 1 - \frac{\sum_{t=1}^{T^*} \alpha \times q(t, e \times G)}{T^*|\alpha|} \quad (18)$$

The elements of vector  $q(t, e \times G)$  are calculated using the recursive formula Eq. (19), with initial inoperability vector  $q(0)$ .

$$q(t + 1) = q(t) - K(e \times G)[I - A^*] \times q(t), K(e \times G) = \text{diag}(h_\phi + \ln(1 + u_\phi e_\phi G)), \phi = 1, 2, \dots, N \quad (19)$$

The fitness value of each genotype represents the resilience of infrastructure system under a restoration resource allocation strategy. For genotypes that do not meet constraint (17), we use a sufficiently large number  $H$  as the penalty of the unavailable solutions.

$$q_\phi(T') < 0.001, \forall i = 1, 2, \dots, N \quad (20)$$

$T^*$  is the largest rapidity of infrastructure systems. The value of  $q(t)$  is decaying with time, if all elements in vector  $q(t)$  are less than 0.001, the performance of infrastructure systems is considered to be back to its expected level. To simplify the computation, if condition (20) is satisfied at  $T' < T^*$ , we assume

$$\sum_{t=T'+1}^{T^*} \alpha \times q(t, e \times G) = 0 \text{ and } Q = 1 - \frac{\sum_{t=1}^{T'} \alpha \times q(t, e \times G)}{T^*|\alpha|} \quad (21)$$

**Step 3. Selection, crossover, mutation and stopping.** The method of roulette, two-point crossover and random mutation are chosen as rules for the selection, crossover and mutation (Davis, 1991). After above procedure, we choose the superior genotypes according to their fitness values in each generation. The rule for stopping is the convergence of the optimal fitness value between the two generations. When algorithm stops, the genotype corresponding to the minimal fitness value is the optimal solution for the resource allocation model.

When applying the solution method, we set the number of individuals in each generation as  $100 \times N$ , where  $N$  is the number of infrastructure systems, the maximum generation as a number which makes maximal fitness function value in each generation converge and not fluctuate for more than 5 steps.

### 5. Numerical example

To validate the model, this section uses a seven infrastructure systems example as a case study. The example is derived from the data provided by the BEA (the US Bureau of Economic Analysis).

#### 5.1. Data and parameter assumptions

The BEA provides the data of national input and output accounts (I-O accounts), which can be applied to generate the interdependency matrix for nearly 500 industry sectors of the U.S. economy (Lian and Haines, 2006). This study considers the interdependencies among energy and transportation infrastructure systems. Seven systems are selected in this case study (list of names and symbols is provided in

**Table 1**  
Infrastructure systems.

Symbol	Name
OGE	Oil and gas extraction
EPG	Electric power generation, transmission, and distribution
NGD	Natural gas distribution
WSS	Water, sewage and other systems
ART	Air, rail, water and truck transportation
TGT	Transit and ground passenger transportation
STS	Scenic transportation and support activities for transportation

Table 1). The data is from U.S. national I-O accounts for year 2011. The average daily performance (in monetary units) of infrastructure systems is shown in Table 2.

In Table 1, some infrastructure systems are composed of several industry sectors in the I-O account, e.g., the ART includes the air transportation, rail transportation, water transportation and truck transportation. In Table 2, the numbers represent the values of system performance (measured in millions of dollars) flowing from system to system and users. In the I-O account, the total performance output of an infrastructure system is divided into outputs to different industry sectors and output to the final users. In Table 2, entries in each row show the distribution of performance output to an infrastructure system. The values for different systems are set according to the real data. The value of ‘performance output’ in each row is proportional to the total performance output of an infrastructure system in the I-O account. It is calculated by applying Eq. (22).

$$\text{performance output} = \left( \frac{\text{output to chosen infrastructure systems}}{\text{output to all industry sectors}} \right) \times \text{total performance output} \quad (22)$$

The ‘Exogenous Demand’ in each row represents the performance output of an infrastructure system to final user. Similar as the ‘performance output’, the value of ‘Exogenous Demand’ in each row is proportional to the output of an infrastructure system to final users in the I-O account. It can be calculated as the difference between the ‘performance output’ of an infrastructure system and the sum of its outputs to chosen systems.

Entries in a column show the distribution of performance inputs of an infrastructure system. That is, each column shows the performance input of an infrastructure system from chosen infrastructure systems for its performance output. From perspective of economy, the performance input and output of each infrastructure system should be equivalent (Miller and Blair, 1985), so here the performance input of an infrastructure system is derived alternatively from its performance output. The ‘Value Added’ in each column represents the amount of economic value added to infrastructure systems, including the cost of employees and taxes. In the example, the value of ‘Value Added’ in each column is calculated as the difference between the ‘performance input’ of an infrastructure system and the sum of its input from chosen systems.

The performance flows among infrastructure systems in Table 2 are denoted by a  $7 \times 7$  matrix. Dividing each element of the performance flow matrix by the respective column sum yields a matrix  $A$ . In Eq. (10),  $A^*$  is a normalized interdependency matrix, the elements in  $A^*$  represent how much inoperability  $q$  is contributed by the column system to the corresponding row system due to interdependent nature of systems. According to literature (Lian and Haines, 2006), matrix  $A^*$  can be derived by Eq. (23).

$$A^* = \left\{ a_{ij}^* = a_{ij} \left( \frac{\text{output of column } j}{\text{output of column } i} \right) \right\} \quad (23)$$

where  $a_{ij}$  is the elements in matrix  $A$ , output of column  $i$  and  $j$  represent the performance output of infrastructure system  $i$  and  $j$ .

Suppose infrastructure system OGE, EPG, and NGD are directly impacted by a disruptive event at  $t = 0$ . The government decision maker is a hypothetical one and responsible for maximizing the resilience of infrastructure systems. In this case, the resources are assumed to be allocated to the directly impacted infrastructure systems. Table 3 displays the parameters in the model.

The basic restoration capacity is taken from the literature (Lian and Haines, 2006; MacKenzie et al., 2016), in which the parameters of most industry sectors are estimated to belong to [0.01, 0.3]. Since big restoration resource budget is needed for a large-scale disruptive event, the cost-effectiveness parameters should be relatively small. The effectiveness of allocating resources to different industry sectors is estimated by MacKenzie et al. (2016) according to the return on investment

**Table 2**  
Average daily performance of infrastructure systems (in millions of dollars).

	OGE	EPG	NGD	WSS	ART	TGT	STS	Exogenous demand	Performance output
OGE	1.285	2.489	3.237	0	0	0.16	0	2.957	10.128
EPG	0.759	1.725	3.200	0.156	0.798	0.264	0.287	4.115	11.304
NGD	2.078	1.790	0.380	2.266	0.784	0.138	0.914	5.110	13.460
WSS	0.444	1.079	0.801	0.182	0.330	0.295	0.101	3.791	7.023
ART	0.502	0.807	0.407	0.261	2.501	0.44	1.492	1.639	8.049
TGT	0	0.145	0.067	0.128	0.192	0	2.224	2.277	5.033
STS	0.481	2.203	0.478	0.06	2.260	0.146	2.025	6.557	14.210
Value Added	4.579	1.066	4.890	3.970	1.184	3.590	7.167		
Performance Input	10.128	11.304	13.460	7.023	8.049	5.033	14.210		

for each industry sector, such as fishing industry, accommodations industry, and oil and gas industry. The effectiveness parameters for these industries are very small, not more than 0.08 per \$1 million. This study assumes the restoration resources are only allocated to infrastructure systems which are directly impacted. The cost-effectiveness parameters for these infrastructure systems are assumed, and for other systems they are set as 0. The expected performance of a specific infrastructure system is equivalent to its performance output in monetary units (see Table 2).

5.2. Results

As the performance output of infrastructure systems give priority to the demands of users,  $c^*(t) = 0$  in Eq. (10). Transforming Eq. (10) into discrete-time dynamic version, substituting the expression of  $K = \text{diag}(k_1, \dots, k_N)$  and  $A^*$  into the equation, the recursive formula of inoperability  $q(t)$  becomes

$$(q(t + 1) = (I - K) \times q(t) + K \times A^* \times q(t), q(0) = (0.3, 0.1, 0.2, 0, 0, 0, 0)^T \tag{24}$$

where the time step is assumed to be 1 day. According to Eq. (24), without restoration resources from government, the change of inoperability  $q(t)$  for infrastructure systems following the disruptive event is shown in Fig. 2.

In Fig. 2, for directly impacted infrastructure systems, the inoperability of OGE and NGD decreases with time because of their basic restoration capacity. However, the inoperability of infrastructure EPG first increases and then decreases. The cause of this behavior is due to the higher direct impacts on infrastructure OGE and NGD. The inoperability of infrastructure EPG increases because of the indirect impacts from infrastructure OGE and NGD. Also, Fig. 2 shows that, due to interdependencies, the other four infrastructure systems are similarly indirectly impacted. Their inoperability first increases and then decreases, while the disturbances of these systems are relatively small. The biggest inoperability during the process is less than 0.04. After about 80 days, the inoperability for every infrastructure system is less than 0.01, so we set 80 days as the largest rapidity  $T^*$ . According to Eq. (4), without restoration resources, the resilience of systems is 0.9663.

Considering the available restoration resources, Fig. 3 depicts the resilience of infrastructure systems by solving the resource allocation model for budgets ranging from 0 to 100 million dollars. It can be seen that, the resilience increases with increase of budget, from 0.9663 at budget 0, to more than 0.99 at budget \$100 million. However, the

**Table 3**  
Parameters in the model.

Infrastructure	OGE	EPG	NGD	WSS	ART	TGT	STS
Initial inoperability $q_\phi(0)$	0.3	0.1	0.2	0	0	0	0
Basic restoration capacity $h_\phi$	0.1	0.1	0.15	0.1	0.1	0.1	0.1
Cost-effectiveness parameter $u_\phi$ (per \$1 million)	$1 \times 10^{-2}$	$1.5 \times 10^{-2}$	$2 \times 10^{-2}$	0	0	0	0
Expected performance $\alpha_\phi$ (in million dollars)	10.128	11.304	13.46	7.023	8.049	5.033	14.21

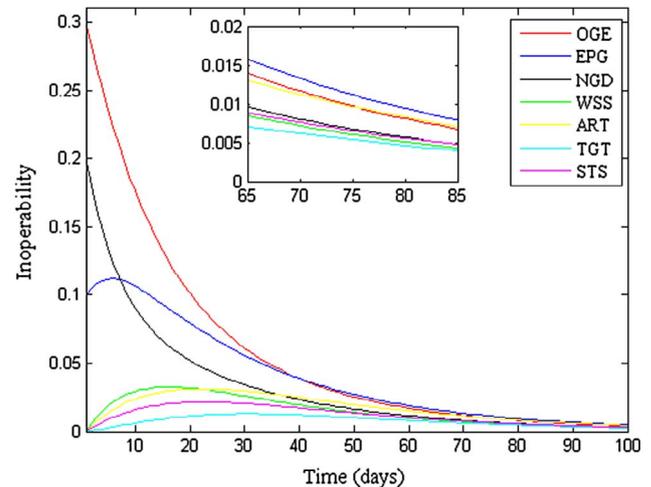


Fig. 2. The Change of inoperability of infrastructure systems.

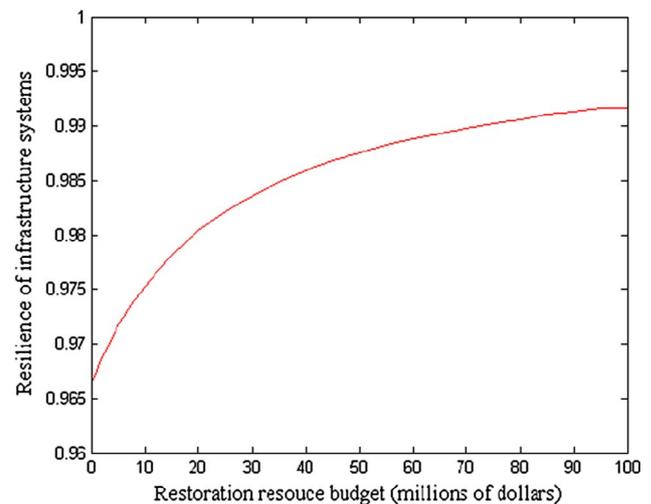


Fig. 3. Resilience of systems at different budget.

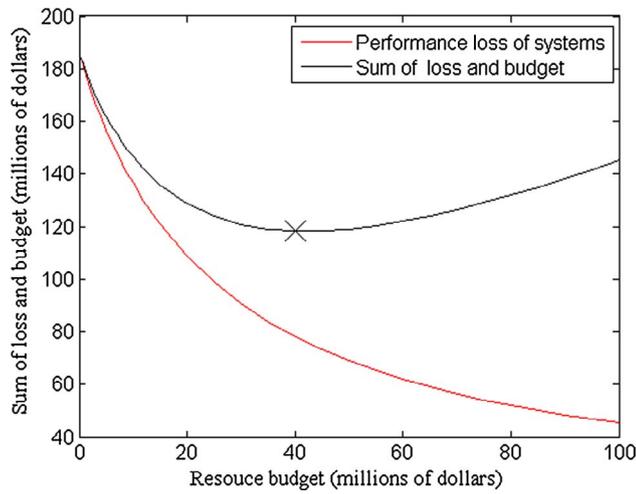


Fig. 4. The sum of loss and budget at different budget.

change of resilience is showing marginal decrease. The total performance loss of infrastructure systems (measured in millions of dollars) at different resource budget levels is shown in Fig. 4, the loss decreases with increase of the budget. Further, we examine the sum of performance loss and restoration resource budget. The sum first decreases and then increases when restoration resource budget increases. It means if we consider the cost of restoration resources and the performance loss resulted by the event as a whole, there is an optimal value for the budget, which minimizes the sum of the two items. In this example, the optimal budget is \$40.46 million, the corresponding resilience of infrastructure systems is 0.9861, the sum of performance loss and resource budget is \$121.193 million.

Next, we investigate the resource allocation strategy at different resource budget levels. Table 4 shows the optimal allocation to infrastructure systems at five budget levels: 1, 5, 10, 50 and 100 million dollars.

In Table 4, if the resource budget is small, e.g. 1 or 5 million dollars, no resource will be allocated to infrastructure EPG, and the biggest proportion of the resource should be spent on infrastructure NGD. In the simulation, the resource allocation to infrastructure EPG is always 0 if the resource budget is below \$6.5 million; infrastructure NGD receives the biggest budget share when the budget is less than \$5.5 million. If the resource budget is large, e.g. \$10, \$50 and \$100 million, the decision maker should allocate the biggest budget share to infrastructure OGE, and the budget share for infrastructure EPG remain the smallest. The optimal resource allocation strategies at different budget levels are determined by the value of parameters in Table 3 and matrix  $A^*$ . The results in Table 4 are probably caused by the fact when the budget is small, the cost-effectiveness parameter  $u_\phi$  plays the main role in resource allocation. Bigger cost-effectiveness parameter makes resources more effective for the restoration of a system. Therefore, infrastructure OGD requires the biggest share of the resources. The effectiveness parameter for infrastructure EPG is in the middle, while the initial inoperability of EPG is the smallest, so resource allocation to EPG

Table 4  
Optimal allocation at different resource budget levels.

Infrastructure	Amount of resources allocated to infrastructure system				
OGE	0.262	2.267	4.652	20.838	41.805
EPG	0	0	1.38	12.215	24.03
NGD	0.738	2.833	3.968	16.947	34.165
Total budget (million dollars)	1	5	10	50	100
Resilience	0.9676	0.9717	0.9753	0.9875	0.9916

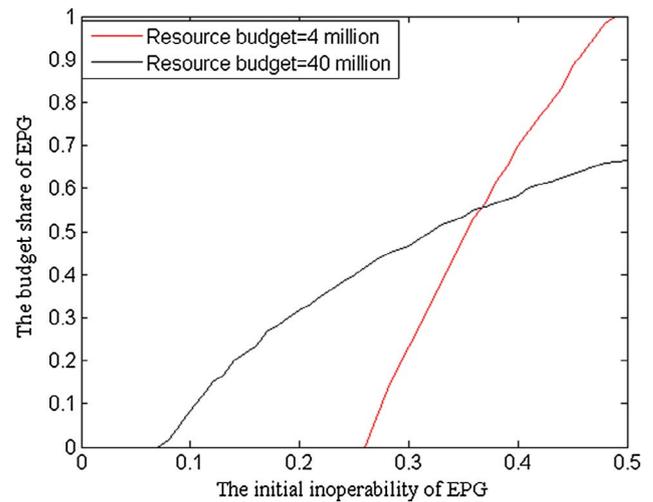


Fig. 5. The budget share of infrastructure EPG with different initial inoperability.

is smaller than to others. If the resource budget is larger, the value of initial inoperability plays the main role in resources allocation, so the rank of budget share from big to small is always OGE, NGD and EPG. It can also be seen that, when the budget increases from \$50 to \$100 million, the change in resilience of infrastructure systems is relatively small due to the marginal decreasing rate of resilience with increase of resources.

### 5.3. Sensitivity analysis

Sensitivity analysis on parameters provides insight into how these parameters affect the optimal allocation of resources.

First, we explore the sensitivity of the optimal allocation to the initial inoperability of infrastructure EPG. One result for resource allocation model recommends allocating smallest share of budget to infrastructure EPG at any resource budget level due to its small initial inoperability. Sensitivity analysis can reveal if this recommendation remains valid if the initial inoperability of infrastructure EPG changes. We tested \$4 million and \$40 million as the resource budget. Fig. 5 shows the budget share of infrastructure EPG with initial inoperability ranging from 0 to 0.5. The initial inoperability for infrastructure OGE and NGD are still 0.3 and 0.2.

Fig. 5 shows that, at two different resource budgets, to maximize resilience of infrastructure systems, the budget shares of infrastructure EPG are both 0 if its initial inoperability is small, and monotonically increases with the increase of initial inoperability above a specific threshold. The threshold for budget \$4 million is larger than that for budget \$40 million. It means that, if the initial inoperability is small, the initial inoperability plays less significant role in resources allocation under a small resource budget, same as results derived from Table 4. However, if the initial inoperability is large enough, the increasing of the budget share of infrastructure EPG at small budget is more rapid. In Fig. 5, the budget share reaches to 1 when the initial inoperability is near 0.5 for budget of \$4 million (this extreme level of initial inoperability is very unlikely in reality). In comparison, the budget share is only 0.66 for budget of \$40 million with the same initial inoperability. This is complementing the simulation results. That is, to maximize infrastructure systems resilience, if the initial inoperability of a system is small, it will require a smaller budget share with a small resources budget; while, if the initial inoperability is large enough, the system will require a larger budget share with a small budget.

One key result from the simulation is that if the resource budget is small, the cost-effectiveness parameter plays the main role in resource allocation. However, the sensitivity analysis on initial inoperability shows that the initial inoperability is also important for resource

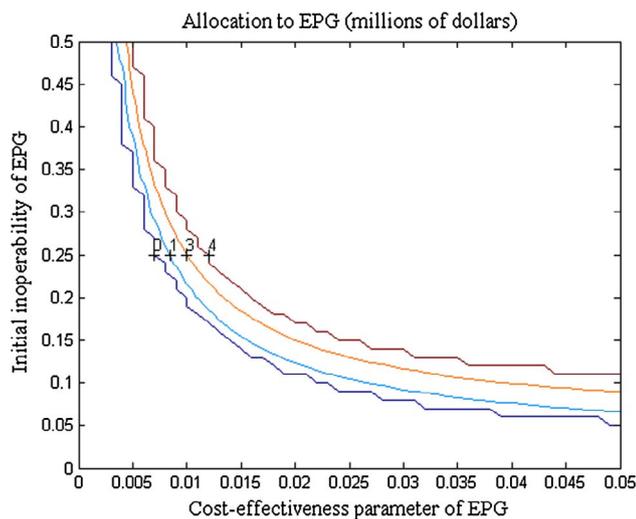


Fig. 6. Contour plot of allocation to EPG with budget \$4 million.

allocation. Sensitivity analysis can reveal the effect of the two parameters on the allocation of resources. Assuming the budget is \$4 million, Fig. 6 is a contour plots of resource allocation to infrastructure EPG with its cost-effectiveness parameter  $u_2$  ranging from 0 to  $5 \times 10^{-2}$  and initial inoperability from 0 to 0.5.

In Fig. 6, with the objective of maximizing resilience, the optimal allocation to infrastructure EPG will increase if its initial inoperability or effectiveness parameter  $u_2$  increases. However, when the initial inoperability is smaller than 0.05, even when we choose  $5 \times 10^{-2}$  as the cost-effectiveness parameter, the allocation to infrastructure EPG is still 0. This result is similar to that shown in Table 4, that infrastructure EPG always obtains the smallest budget share because of its small initial level of inoperability. Similarly, if the cost-effectiveness parameter is smaller than  $5 \times 10^{-3}$ , the allocation to infrastructure EPG will always be 0, no matter which initial inoperability level is selected. Comparing the effect of the two parameters on allocation, in Fig. 6, if the initial inoperability is chosen as 0.2 or larger, the optimal allocation to infrastructure EPG will be very sensitive to its cost-effectiveness parameter. As the cost-effectiveness parameter increases from  $1 \times 10^{-2}$  to  $1.5 \times 10^{-2}$ , the allocation to infrastructure EPG will increase from 0 to 4. This means that, for an infrastructure system with large initial inoperability, the allocation is more sensitive to the value of cost-effectiveness parameter. With similar analysis, it can be found that, the optimal allocation will become less sensitive to the initial inoperability with larger cost-effectiveness parameter. In summary, the sensitivity of the optimal resource allocation to cost-effectiveness parameter is closely related to the initial inoperability.

## 6. Conclusions

Enhancing resilience can reduce the impacts of disruptive events on interdependent infrastructure systems. This paper presented an optimization model to assist decision makers in determining the effective resource allocation and maximize resilience of interdependent infrastructure systems. The model and the application in this study can guide the decision making process that will result in enhancing interdependent infrastructure resilience to disruptive events. First, though larger resource budget can result in higher resilience, the change in resilience is marginally decreasing with the increase of the recovery budget. There is an optimal value for the budget, which can minimize the sum of restoration costs and the performance loss (measured in monetary units) induced by a disruptive event. Second, under a disruptive event, the budget share of interdependent infrastructure systems will be different if the amount of resource budget changes. Finally, with the same resource budget, the budget share of an infrastructure

system is sensitive to its initial inoperability and cost-effective parameter. The sensitivity to one parameter is closely related to the value of the other.

There are also some limitations of the study. First, above recommendations are dependent on the modeling assumptions, such as: (i) the logarithmic relationship between the allocated resources and the recovery rate of an infrastructure system; (ii) linear DIIM model used to describe the interaction process among infrastructure systems; etc. Although we believe these assumptions, to some extent, illustrate the characteristics of infrastructure systems, their validity still needs to be explored due to the complexity of real infrastructure systems. Second, the impacts of factors such as basic restoration capacity, restoration-starting time, and resource classification and so on are not taken into consideration in this study. In the future work, empirical studies will be conducted to validate some of assumptions in this study, or the interaction effects of multiple factors on restoration resource allocation will be investigated.

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